Precalc Warm Up # 10-2

1. Find the sum of the sequence

   {9, 7, 5, …, -131}

2. $a_n = 12 - 3n$

   Find S_{100}

HW Questions: p. 248

EXERCISES 8.1.2

1. Find the sum of the first ten terms in the arithmetic sequences
(a) $\{1, 4, 7, 10, \ldots\}$ (b) $\{3, 9, 15, 21, \ldots\}$ (c) $\{10, 4, -2, \ldots\}$.

2. For the given arithmetic sequences, find the sum, S_n, to the requested number of terms.

(a) $\{4, 3, 2, \ldots\}$ for $n = 12$

(b) $\{4, 10, 16, \ldots\}$ for $n = 15$

(c) $\{2.9, 3.6, 4.3, \ldots\}$ for $n = 11$

3. Find the sum of the following sequences:

(a) $\{5, 4, 3, \ldots, -15\}$

(b) $\{3, 9, 15, \ldots, 75\}$

(c) $\{3, 5, 7, \ldots, 29\}$

$a_n = a_1 + d(n-1)$

$75 = 3 + 6(n-1)$

$75 = -3 + 6n$

$78 = 6n$

$n = 13$

$S_n = \frac{n}{2}[2a_1 + d(n-1)]$

$S_{13} = \frac{13}{2}[2(3) + 6(13-1)]$

4. The weekly sales of washing machines from a retail store that has just opened in a new housing complex increases by 2 machines per week. In the first week of January 1995, 24 machines were sold. $d = 2$

(a) How many are sold in the last week of December 1995?

(b) How many machines did the retailer sell in 1995?

(c) When was the 500th machine sold?

n	1	2	3
	24	26	28

$$a_{52} = 24 + 2(51)$$

b) $$S_{52} = \frac{52}{2}\left[2(24) + 2(51)\right]$$

4. The weekly sales of washing machines from a retail store that has just opened in a new housing complex increases by 2 machines per week. In the first week of January 1995, 24 machines were sold. $d = 2$

(a) How many are sold in the last week of December 1995?

(b) How many machines did the retailer sell in 1995?

(c) When was the 500th machine sold?

n	1	2	3
	24	26	28

n

c) $$500 = \frac{n}{2}\left[2(24) + 2(n-1)\right]$$

$$1000 = n(48 + 2n - 2)$$

$$= n(46 + 2n)$$

$$1000 = 46n + 2n^2$$

$$0 = 2n^2 + 46n - 1000$$

$$0 = n^2 + 23n - 500$$

$$n = \frac{-23 \pm \sqrt{23^2 - 4(-500)}}{2}$$

$n \approx 13.6$ ~~-36.6~~

by the 14th week

5. The fourth term of an arithmetic sequence is 5 while the sum of the first 6 terms is 10 Find the sum of the first nineteen terms.

$a_4 = 5$ $\quad S_6 = 10$ $\quad S_{19} = ?$

$5 = a_1 + d(3)$ $\quad 10 = \frac{6}{2}[2a_1 + d(5)]$

$a_1 + d$

6. Find the sum of the first 10 terms for the sequences defined by

(a) $u_n = -2 + 8n$ (b) $u_n = 1 - 4n$

$a_1 = -2 + 8(1)$

$= 6$

$a_2 = -2 + 8(2)$

$= 14$

$d = 8$

$S_{10} =$

7. The sum of the first eight terms of the sequence $\{\ln x, \ln x^2y, \ln x^3y^2, \ldots\}$ is given by $S_8 = 4(a\ln x + b\ln y)$. Find a and b.

a_1 a_2 a_3

$d = \ln x^2y - \ln x$ Now condense

$d = \ln\left(\frac{x^2y}{x}\right)$

$d = \ln xy$

$S_n = \frac{n}{2}[2(\ln x) + (\ln xy)(n-1)]$

$S_8 = \frac{8}{2}[2\ln x + 7\ln xy]$ (Need to split these up)

$S_8 = 4[2\ln x + 7(\ln x + \ln y)]$

$S_8 = 4[2\ln x + 7\ln x + 7\ln y]$

$S_8 = 4(9\ln x + 7\ln y)$

a b

8.1.1 Arithmetic Sequence: $a_n = a_1 + d(n-1)$

8.1.2 Arithmetic Series:
the sum of an arithmetic sequence

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$= \frac{n}{2}(a_1 + a_1 + d(n-1))$$

$$= \frac{n}{2}(2a_1 + d(n-1))$$

Today 8.1.3 Sigma Notation: Sum

$\sum_{i=2}^{34} i$ means $2 + 3 + 4 + \ldots + 34$

$n = 34 - 2 + 1$

$= 33$ terms

$S_{33} = \frac{33}{2}(2 + 34)$

$= 594$

Find $\sum_{i=1}^{50}(2i+3) =$

$n = 50 - 1 + 1$
$= 50$ terms

$a_1 = 2(1)+3 = 5$
$a_2 = 2(2)+3 = 7$
$a_3 = 2(3)+3 = 9$
$\Big\} d = 2$

$S_{50} = \frac{50}{2}[2(5) + 2(49)]$
$= 25(108)$
$= 2700$

$\sum_{k=0}^{15}(-4k+5) =$ (d)

$n = 15 - 0 + 1$
$= 16$

$S_{16} = \frac{16}{2}[2(5)+(-4)(15)]$
$= 8(10-60)$
$S_{16} = -400$

$a_1 = -4(0)+5$
$a_2 = -4(1)+5$
$= 1$

$$\sum_{x=3}^{18} 2 = 2 + 2 + 2 + \ldots = 16(2) = 32$$

$n = 18 - 3 + 1$
$= 16$

Property
if k is a constant

$$\sum_{x=1}^{n} k = kn$$

Groups: find $\sum_{t=6}^{84} (7 - 3t)$

$d = -3$

$n = 84 - 6 + 1$

$a_1 = 7 - 3(6) = -11$

$a_2 = 7 - 3(7) =$

$a_3 =$

$$S_{79} = \frac{79}{2}\left[2(-11) + (-3)(78)\right]$$

Groups: find $\sum_{t=6}^{84}(7-3t)$

$a_1 = 7-3(6) = -11$

$a_2 = 7-3(7) = -14$

$a_3 = 7-3(8) = -17$

$d = -3$

$n = 84-6+1$

$n = 79$ terms

$$S_{79} = \frac{79}{2}\left[2(-11)+(-3)(78)\right]$$

$$S_{79} = \frac{79}{2}(-22-234)$$

$$S_{79} = -10112$$

IMPORTANT:

When solving word problems involving sequences,

A car's original value was $23,000 and decreases in value by $80 per month. How many months will it take before the value drops below $15000?

$d = -80$

0	after 1 mo	after 2 mo
23,000	a_1	

Value < 15000

A car's original value was $23,000 and decreases in value by $80 per month. How many months will it take before the value drops below $15000?

0	after 1 mo	after 2 mo
23,000	22,920	22,840

$d = -80$

a_1 a_n

Value < 15000

$a_1 + d(n-1) < 15000$

A car's original value was $23,000 and decreases in value by $80 per month. How many months will it take before the value drops below $15000?

0	after 1 mo	after 2 mo
23,000	22,920	22,840

$d = -80$

a_1 a_n

Value < 15000

$a_1 + d(n-1) < 15000$

$22,920 - 80(n-1) < 15000$

$n > 100$

101 months

TJ is starting a t-shirt company. He predicts that he will sell 20 shirts in the first month, 23 in the second month, 26 in the third and so on, in arithmetic sequence. When can he expect to sell his thousandth t-shirt? n

$a_1 = 20 \quad a_2 = 23 \quad d = 3$

$1000 = \frac{n}{2}\left[2(20) + 3(n-1)\right]$

$2000 = n(40 + 3n - 3)$

$0 = 3n^2 + 37n - 2000$

$n = \frac{-37 \pm \sqrt{37^2 - 4(3)(-2000)}}{2(3)}$

$= \frac{-37 \pm \sqrt{1369 + 24000}}{6}$

$= \frac{-37 \pm \sqrt{25369}}{6}$

$n \approx 20.4$

so by the 21st month.

HW: SL Book

p. 250 #1-9, 12, 13b

#11 is a fun problem! I challenge you to try! It won't be on a HM quiz, the solution will be on line at the end of Thursday's lesson.